Poverty Measurement and the Distribution of Deprivations among the Poor

Sabina Alkire

OPHI, Oxford

James E. Foster

George Washington University and OPHI, Oxford

UNU-WIDER Conference on 'Inequality - measurement, trends, impacts, and policies'

Helsinki, 5-6 September 2014

Introduction

Two forms of technologies for evaluating poverty

- identification and aggregation of Sen (1976)

1 Unidimensional methods apply when:

- **Single** welfare variable – eg, calories
- Variables can be combined into **one** aggregate variable – eg, expenditure

2 Multidimensional methods apply when:

- Variables **cannot** be meaningfully aggregated – eg, sanitation conditions and years of education
- Desirable to leave variables **disaggregated** because sub-aggregates are policy relevant – eg food and nonfood consumption

Introduction

- Recently, strong **demand** for tools for measuring poverty multidimensionally
  - Governments, international organizations, NGOs
- Literature has responded with new measures
  - Anand and Sen (1997)
  - Tsui (2002)
  - Atkinson (2003)
  - Bourguignon and Chakravarty (2003)
  - Deutsch and Silber (2005)
  - Chakravarty and Silber (2008)
  - Maasoumi and Lugo (2008)

Introduction

- Problems
  - Most inapplicable to **ordinal** variables
    - Encountered in poverty measurement
  - Or yield methods that are far too **crude**
    - Violate Dimensional Monotonicity
    - Non-discerning identification: Very few poor or very few nonpoor

Introduction

- Methodology introduced in Alkire-Foster (2011)
  - Identification: Dual cutoff z and k
  - Measure: Adjusted headcount ratio $\mathbf{M_0}$
- Addressed these problems
  - Applies to **ordinal**
    - And even categorical variables
  - Not so **crude**
    - Satisfies Dimensional Monotonicity
    - Discerning identification: not all poor or all nonpoor
- Satisfies key properties for policy and analysis
  - **Decomposable** by population
  - **Breakdown** by dimension after identification

Introduction

Specific implementations include:

- **M**ultidimensional **P**overty **I**ndex (UNDP)
  - Cross country implementation of M_0 by OPHI and HDRO
- Official poverty index of **Colombia**
  - Country implementation of M_0 by Government of Colombia
- **G**ross **N**ational **H**appiness index (Bhutan)
  - Country implementation of $(1-M_0)$ by Center for Bhutan Studies
- **W**omen's **E**mpowerment in **A**griculture **I**ndex (USAID)
  - Cross country implementation of $(1-M_0)$ by USAID, IFPRI, OPHI

Introduction

- One possible critique
  - M_0 is **not sensitive enough to distribution among the poor**
- Two forms of distribution sensitivity among poor
  - To inequality **within dimensions**
    - Kolm (1976)
  - To positive association **across dimensions**
    - Atkinson and Bourguignon (1982)
- Many existing measures satisfy one or both
  - Adjusted FGT of Alkire-Foster (2011)
  - However, adjusted FGT not applicable to **ordinal** variables

This Paper

Asks

Can M_0 be altered to obtain a method that is both

- **sensitive to distribution** among the poor
- and applicable to **ordinal data**?

Answer

Yes. In fact, as easy as constructing unidimensional measures satisfying the transfer principle

Key

Intuitive transformation from **unidimensional** to **multidimensional** measures

Offers insight on the structure of M_0 and related measures

This Paper

- However we lose
  - **Breakdown** by dimension after identification
- Question
  - Is there any multidimensional measure that is sensitive to the distribution of deprivations and also can be broken down by dimension?
- Answer
  - Classical **impossibility** result
  - Can have one **or** the other but **not both**!
- Bottom line
  - Recommend using M_0 with an associated inequality measure

Outline

- Poverty Measurement
  - Unidimensional
  - Multidimensional
- Transformations
  - Measures
  - Axioms
- Impossibilities and Tradeoffs
- Conclusions

Poverty Measurement

- Traditional framework of Sen (1976)
- Two steps
  - Identification: "Who is poor?"
    - Targeting
  - Aggregation "How much poverty?"
    - Evaluation and monitoring

Unidimensional Poverty Measurement

Typically uses **poverty line** for identification

- Early definition: Poor if income below or equal to cutoff
- Later definition: Poor if income strictly below cutoff
  - Example: Income distribution x = (7,3,4,8) poverty line $\pi = 5$
  - Who is poor?

Unidimensional Poverty Measurement

- Typically uses **poverty measure** for aggregation
  - Formula aggregates data to poverty level
    - Examples: Watts, Sen
    - Example: FGT
      - Where: g_i^α is $[(\pi - x_i)/\pi]^\alpha$ if i is poor and 0 if not, and $\alpha \geq 0$ so that
        - $\alpha = 0$ headcount ratio
        - $\alpha = 1$ per capita poverty gap
        - $\alpha = 2$ squared gap, often called FGT measure

$$P_\alpha(x;\pi) = \mu(g_1^\alpha, \ldots, g_n^\alpha) = \mu(g^\alpha)$$

Unidimensional Poverty Measurement

Example

Incomes $x = (7,\underline{1},\underline{4},8)$

Poverty line $\pi = 5$

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0(x;\pi) = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 4/5, 1/5, 0)$

Poverty gap = HI = $P_1(x;\pi)$ $= \mu(g^1) = 5/20$

Squared gap vector $g^2 = (0, 16/25, 1/25, 0)$

FGT Measure = $P_2(x;\pi)$ $= \mu(g^2) = 17/100$

Unidimensional Poverty Measurement

FGT Properties

- For $\alpha = 0$ (headcount ratio)
  - Invariance Properties: Symmetry, Replication Invariance, Focus
  - Composition Properties: Subgroup Consistency, Decomposability
- For $\alpha = 1$ (poverty gap)
  - +Dominance Property: Monotonicity
- For $\alpha = 2$ (FGT)
  - +Dominance Property: Transfer

Unidimensional Poverty Measurement

- Poverty line actually has **two** roles
  - In identification step, as the separating **cutoff** between the target group and the remaining population.
  - In aggregation step, as the **standard** against which shortfalls are measured
- In some applications, it may make sense to **separate** roles
  - A **poverty standard** π_A for constructing gap and aggregating
  - A **poverty cutoff** $\pi_I \leq \pi_A$ for targeted identification
    - Example 1: Measuring ultra-poverty Foster-Smith (2011)
      - Forcing standard π_A down to cutoff π_I distorts the evaluation of ultrapoverty
    - Example 2: Measuring hybrid poverty Foster (1998)
- Broader class of poverty measures $P(x; \pi_A, \pi_I)$

Unidimensional Poverty Measurement

Example: FGT $P_\alpha(x; \pi_A, \pi_I)$

Incomes $x = (7,\underline{1},4,8)$

Poverty standard $\pi_A = 5$

Poverty cutoff $\pi_I = 3$

Deprivation vector $g^0 = (0,1,0,0)$ *(use π_I for identification)*

Headcount ratio $P_0(x; \pi_A, \pi_I) = \mu(g^0) = 1/4$

Normalized gap vector $g^1 = (0, 4/5, 0, 0)$ *(use π_A for gap)*

Poverty gap = *HI* = P_1 $(x; \pi_A, \pi_I) = \mu(g^1) = 4/20$

Squared gap vector $g^2 = (0, 16/25, 0, 0)$

FGT Measure = $P_2(x; \pi_A, \pi_I) = \mu(g^2) = 16/100$

Unidimensional Poverty Measurement

- All properties are easily generalized to this environment
- FGT Properties
  - For $\alpha = 0$ (headcount ratio)
    - Invariance Properties: Symmetry, Replication Invariance, and Focus
    - Composition Properties: Subgroup Consistency, Decomposability,
  - For $\alpha = 1$ (poverty gap)
    - +Dominance Property: Monotonicity
  - For $\alpha = 2$ (FGT)
    - +Dominance Property: Transfer

Unidimensional Poverty Measurement

- Idea of poverty measure $P(x;\pi_A,\pi_I)$
  - Allows flexibility of targeting group below poverty cutoff π_I while maintaining the poverty standard at π_A
  - Particularly helpful when different groups of poor have different characteristics and hence need different policies

Multidimensional Poverty Measurement

- How to evaluate poverty with many dimensions?
- Previous work mainly focused on **aggregation**
- While for the **identification** step it:
  - First set cutoffs to identify deprivations
  - Then identified poor in one of three ways
    - Poor if have *any* deprivation
    - Poor if have *all* deprivations
    - Poor according to some function left unspecified
- Problem
  - First two are **impractical** when there are many dimensions
    - Need intermediate approach
  - Last is **indeterminate**, and likely **inapplicable** to ordinal data

AF Methodology

- Alkire and Foster (2011) methodology addresses these problems
- It specifies an **intermediate** identification method that is consistent with **ordinal** data
- **Dual cutoff** identification
  - Deprivation cutoffs $z_1 \ldots z_j$ one per each of *j* deprivations
  - Poverty cutoff k across aggregate weighted deprivations
- Idea
  - A person is poor if multiply deprived enough
- Example

AF Methodology

Achievement Matrix (say equally valued dimensions)

Dimensions

$$Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \textbf{Persons}$$

$$\boldsymbol{z = (\ 13 \quad 12 \quad 3 \quad 1\ )} \quad \textbf{Cutoffs}$$

AF Methodology

Deprivation Matrix

Censored Deprivation Matrix, k=2

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 1 \end{bmatrix} \Longrightarrow g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

Identification Who is poor?

If poverty cutoff is $k = 2$

Then the two middle persons are poor

Now censor the deprivation matrix

Ignore deprivations of nonpoor

AF Methodology

If data cardinal, construct two additional censored matrices

Censored Gap Matrix

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Censored Squared Gap Matrix

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Aggregation

$$M_\alpha = \mu(g^\alpha(k)) \text{ for } \alpha \geq 0$$

Adjusted FGT M_α is the mean of the respective censored matrix

AF Methodology

Properties

- For $\alpha = 0$ (Adjusted headcount ratio)
  - Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
  - Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement
  - Composition Properties: Subgroup Consistency, Decomposability, Dimensional Breakdown
- For $\alpha = 1$ (Adjusted poverty gap)
  - +Dominance Property: Monotonicity, Weak Transfer
- For $\alpha = 2$ (Adjusted FGT)
  - +Dominance Property: Transfer

AF Methodology

Note

- The poverty measures with $\alpha > 0$ use gaps, hence require **cardinal** data
- Impractical given data quality
- Focus here on measure with $\alpha = 0$ that handles **ordinal** data

Adjusted Headcount Ratio M_0

- Practical and applicable

Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k)) = 3/8$

	Domains				c(k)	c(k)/d	
$g^0(k) =$	0	0	0	0	0		
	0	1	0	1	$\underline{2}$	2 / 4	Persons
	1	1	1	1	$\underline{4}$	4 / 4	
	0	0	0	0	0		

H = multidimensional headcount ratio = 1/2

A = average deprivation share among poor = ¾

Note: Easily generalized to where deprivations have different values v_1, v_2, v_3, v_4 summing to d = 4

Adjusted Headcount Ratio

Properties

- Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
- Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement, a form of Weak Transfer
- Composition Properties: Subgroup Consistency, Decomposability, Dimensional Breakdown

Note

- No transfer property **within dimensions**
  - Requires cardinal variables!
- No transfer property **across dimensions**
  - Here there is some scope

New Property

Recall: Dimensional Monotonicity Multidimensional poverty should rise whenever a poor person becomes deprived in an additional dimension (*cet par*) (AF, 2011)

New: Dimensional Transfer Multidimensional poverty should fall as a result of an association decreasing rearrangement among the poor that leaves the total deprivations in each dimension unchanged, but changes their allocation among the poor.

Adjusted Headcount Satisfies Dimensional Monotonicity, but just violates Dimensional Transfer.

Q/ Are there other related measures satisfying **DT**?

New Measures

- Idea
  - Construct attainment matrix
  - Aggregate attainment values to create attainment count vector
  - Apply a unidimensional poverty measure P to obtain a multidimensional poverty measure M
  - The properties of P are directly linked to the properties of M
  - Perhaps M satisfying dimensional transfer can be found

Attainments

Recall **Achievement** matrix

Dimensions

$$Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$

Persons

$$z = (\mathbf{13} \quad \mathbf{12} \quad \mathbf{3} \quad \mathbf{1})$$

Cutoffs

Attainments

Construct **attainment** matrix (recall equal value case)

1 if person attains deprivation cutoff in a given domain

0 if not

Domains

$$a^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{0} & 1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ 1 & \underline{0} & 1 & 1 \end{bmatrix} \quad \text{Persons}$$

Note

Opposite of the deprivation matrix

Attainments

Counting Attainments (equal value case)

1 if person attains cutoff in a given domain

0 if not

$$a^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{0} & 1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ 1 & \underline{0} & 1 & 1 \end{bmatrix} \qquad a = \begin{matrix} 4 \\ 2 \\ 0 \\ 3 \end{matrix}$$

(Columns: Domains; rows: Persons)

Attainment vector

$$a = (4, 2, 0, 3)$$

Now apply unidimensional poverty measure

Transformations

Define $M_P(x;z) = P(a; \pi_A,\pi_I)$

- where a is the attainment vector associated with x
- P is a unidimensional poverty measure
- M_p called *attainment count measure*
- Process of obtaining M_p from P is called *attainment count transformation*

Example 1

- $P = P_0$ unidimensional headcount ratio, $0 < \pi_I < \pi_A = d$
- Poor identified using $\leq$
- Then M_P is multidimensional headcount ratio H with dual cutoff identification having poverty cutoff $k = d - \pi_I$

Transformations

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

- where a is the attainment vector associated with x
- P is a unidimensional poverty measure
- M_p called *attainment count measure*
- Process of obtaining M_p from P is called *attainment count transformation*

Example 2

- $P = P_1$ unidimensional poverty gap ratio, $0 < \pi_I < \pi_A = d$
- Poor identified using $\leq$
- Then M_P is adjusted headcount ratio M_0 with dual cutoff identification having poverty cutoff $k = d - \pi_I$

Note: This is the **standard AF methodology**

Transformations

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

where *a* is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 3

$P = P_0$ unidimensional headcount ratio, $0 < \pi_I = \pi_A < d$

Poor identified using <

Then M_P is multidimensional headcount ratio H with alternate dual cutoff identification having poverty cutoff $k = d - \pi_I$

Alternate: a person is poor if attainment count exceeds k

Transformations

Define $M_P(x;z) = P(a; \pi_A,\pi_I)$

where a is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 4

$P = P_1$ unidimensional poverty gap ratio, $0 < \pi_I = \pi_A < d$

Poor identified using $<$

Then M_P is adjusted headcount ratio M_0 with alternate dual cutoff identification having poverty cutoff $k = d - \pi_I$

Note: The **Mexican version of the AF methodology**

Transformations

Example 2

Recall $a = (4, 2, 0, 3)$

Identification using $\pi_I = 3$ and $\leq$

Who is poor?

$(4, \underline{2}, \underline{0}, \underline{3})$

Aggregation using $\pi_A = 4$ and poverty gap ratio P_1

Gap vector $g^1 = (0, 2/4, 4/4, 0)$

Then

$P_1 = \int (g^1) = 6/16 = M_0$ AF Methodology

Transformations

Example 4

Recall $a = (4, 2, 0, 3)$

Identification using $\pi_I = 3$ and $<$

Who is poor?

$(4, \underline{2}, \underline{0}, 3)$

Aggregation using $\pi_A = 3$ and poverty gap ratio P_1

Gap vector $g^1 = (0, 1/3, 3/3, 0)$

Then

$P_1 =$ ⌠ $(g^1) = 4/12 =$ Mexican version

Transformations

Note: Properties of M_P depend on properties of P

In particular:

- If P satisfies monotonicity, then M_P satisfies dimensional monotonicity.
- If P satisfies transfer, then M_P satisfies dimensional transfer.

Lesson

- Trivial to construct multidimensional measures sensitive to inequality across deprivations – just use distribution sensitive unidimensional measure and transform

Question

- But at what cost?

Impossibility

Crucial property

Dimensional Breakdown: M can be expressed as an average of dimensional functions (after identification)

Note

The measure associated with P_2 does not satisfy dimensional breakdown

Theorem There is no symmetric multidimensional measure M satisfying both dimensional breakdown and dimensional transfer

Proof

Follows impossibility result in literature.

Impossibility

- Importance of Dimensional Breakdown
  - Policy
    - Composition of poverty
    - Changes over time by indicator
  - Analysis
    - Composition of poverty across groups, time
    - Interconnections across deprivations
    - Efficient allocations
- Conclusion
  - Easy to construct measure satisfying dimensional transfer
  - But at a cost: lose this key element of the toolkit

Concluding Remarks

- Alternative way forward:
  - Apply M_0 class of measures for ordinal data
    - Satisfies dimensional breakdown
  - Construct associated measure of inequality among the poor
- Note
  - P_0 headcount ratio, P_1 poverty gap and FGT P_2 have long been used in concert to analyze the incidence, depth, and distribution of (income) deprivations
  - Analogously, can use H headcount ratio, adjusted headcount ratio M_0 and inequality measure to analyze the incidence, breadth and distributions of deprivations
- With a focus on the measure M_0 and its useful breakdown

Thank you