Ordinal bivariate inequality: concepts and application to child deprivation in Mozambique

Christoffer Sonne-Schmidt

The Danish National Centre for Social Research

Finn Tarp

UNU-WIDER, and University of Copenhagen

Lars Peter Østerdal

University of Southern Denmark

Multidimensional approaches (1)

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Alternative weighting schemes may however alter conclusions.

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Following the seminal work by Atkinson and Bourguignon (1982, 1987) and Bourguignon (1989), recent contributions include Duclos et al. (2006, 2007), Bourguignon and Chakravarty (2003), Crawford (2005), Gravel et al. (2009), Duclos and Batana (2010), Gravel and Mukhopadhyay (2010), and Muller and Trannoy (2011) among others.

Still, these contributions apply conditions that are typically formulated in term of **specified signs on the second or higher order cross-derivatives** of the underlying social welfare functions.

Ordinal approaches

We consider the problem of making social welfare or inequality comparisons between populations in a situation where **only ordinal information is available**

Thus, no assumptions are made about the importance of each dimension, nor about the complementarity/substitutability between the dimensions.

Ordinal multidimensional welfare comparisons

Population B is better off than population A

when B's distribution can be obtained from A's by a finite number of shifts of density from one outcome to another that is better.

This concept is known as the **usual** (**stochastic) order.**

or **First Order Dominance (FOD)**.*

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or **First Order Dominance (FOD)**.*

This concept first characterized Lehmann (1955) and later re-discovered in economics by Levhari et al. (1975).

*In the multidimensional context the term "first order dominance" has been used with different meanings. The lower orthant orderings of the Atkinson and Bourguignon type for welfare comparisons are often referred to as first order dominance criteria in the economics literature.

Illustration: 2 dimensions – 4 outcomes

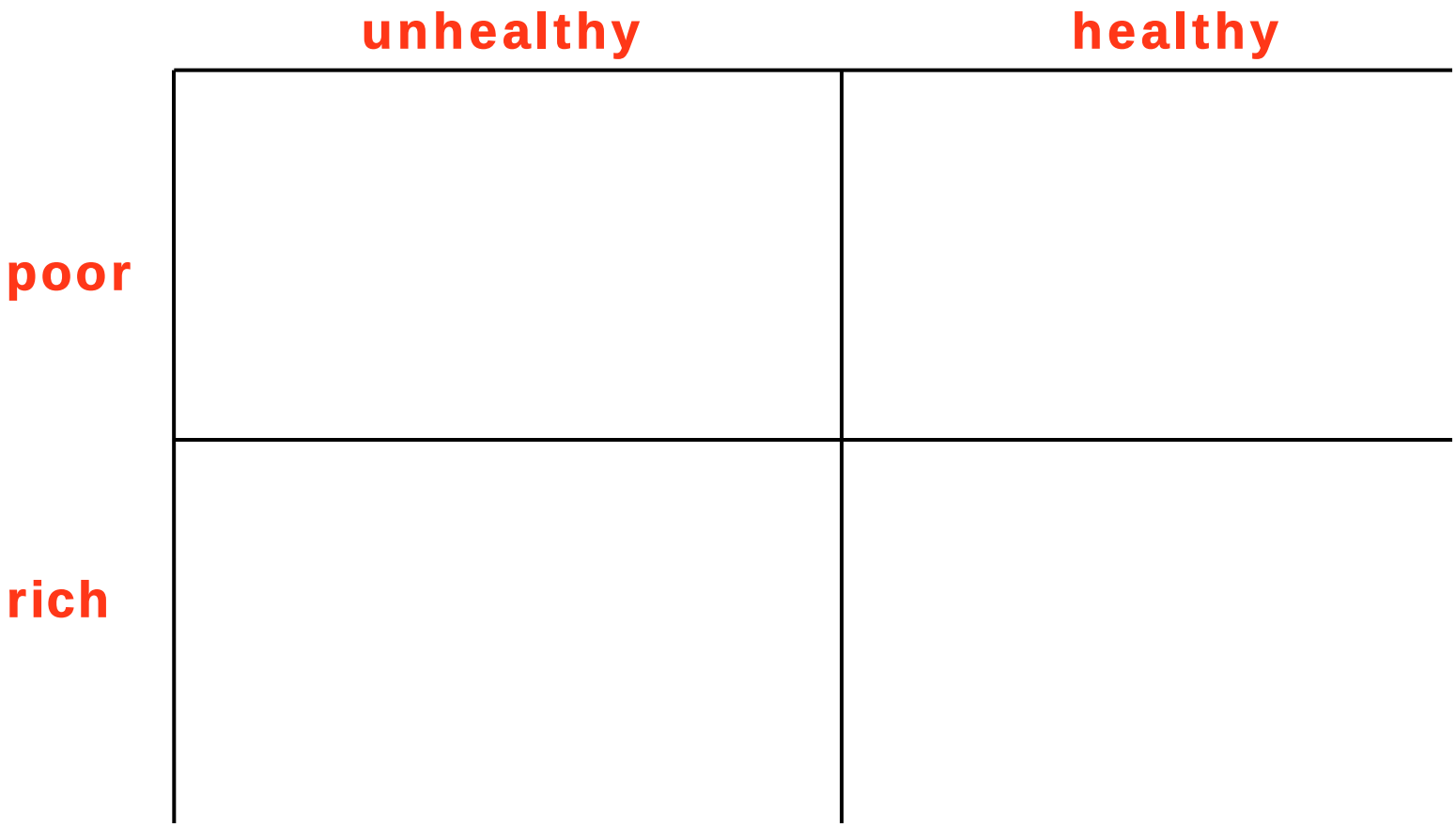
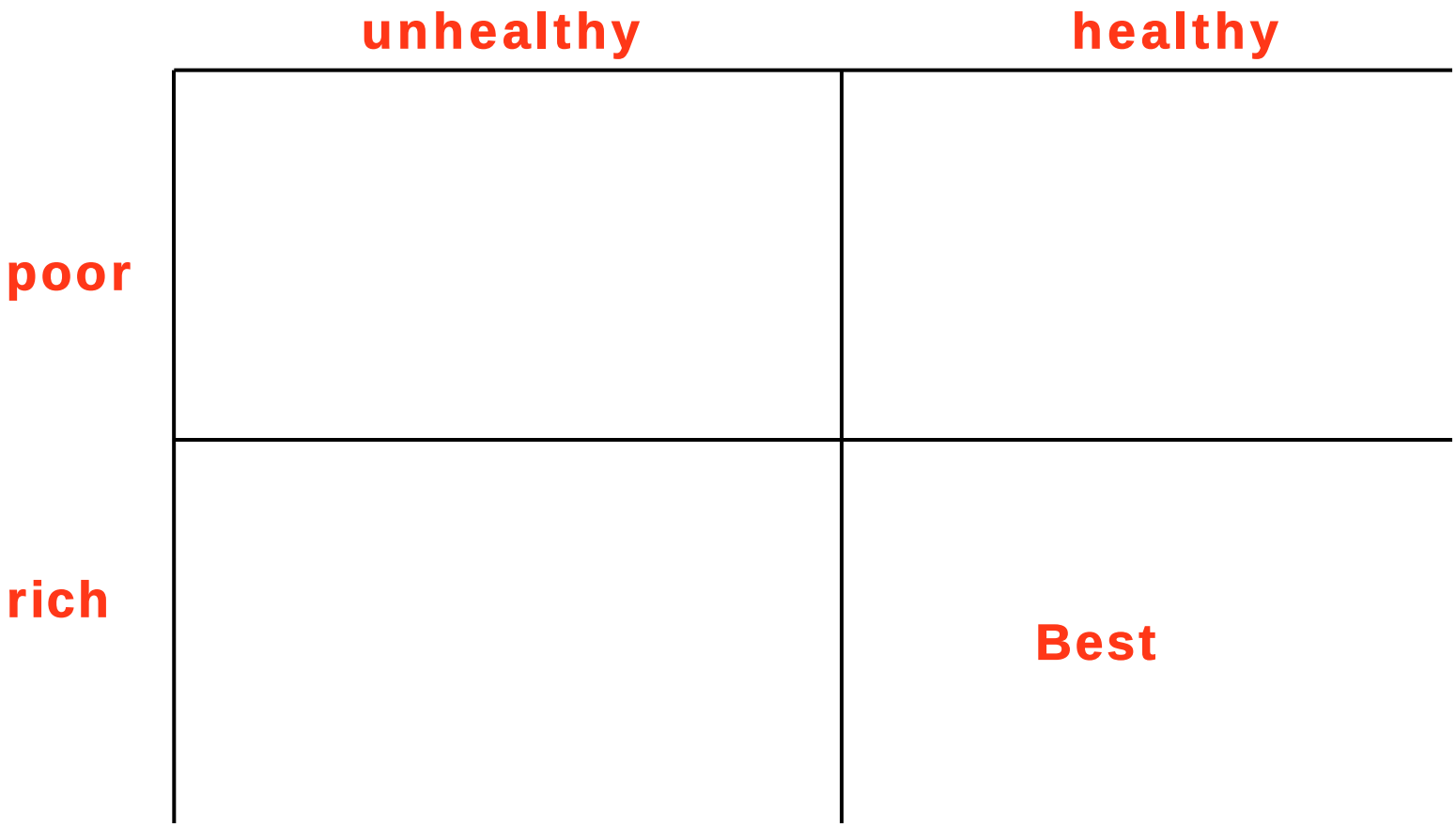


Illustration: 2 dimensions – 4 outcomes



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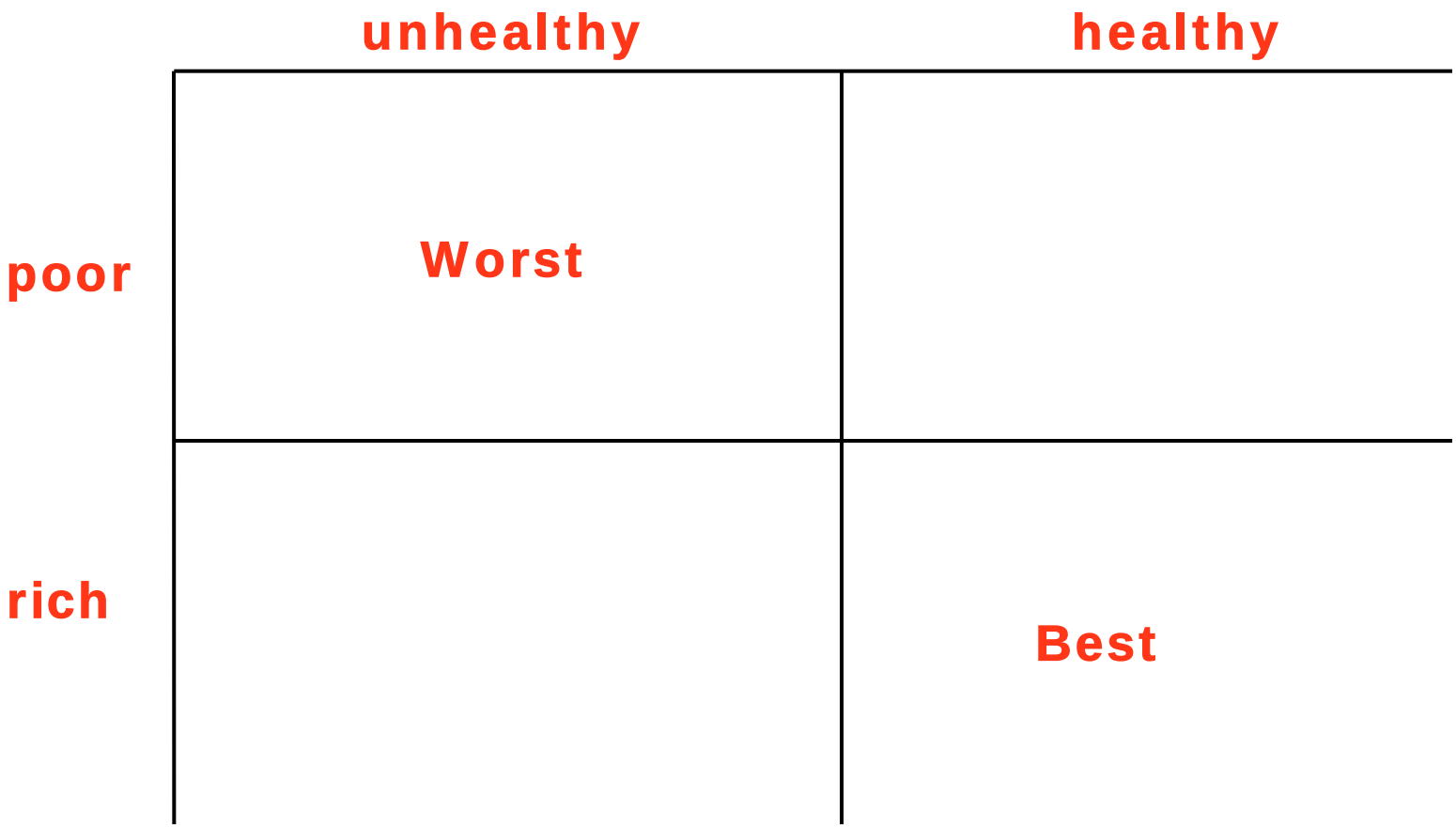
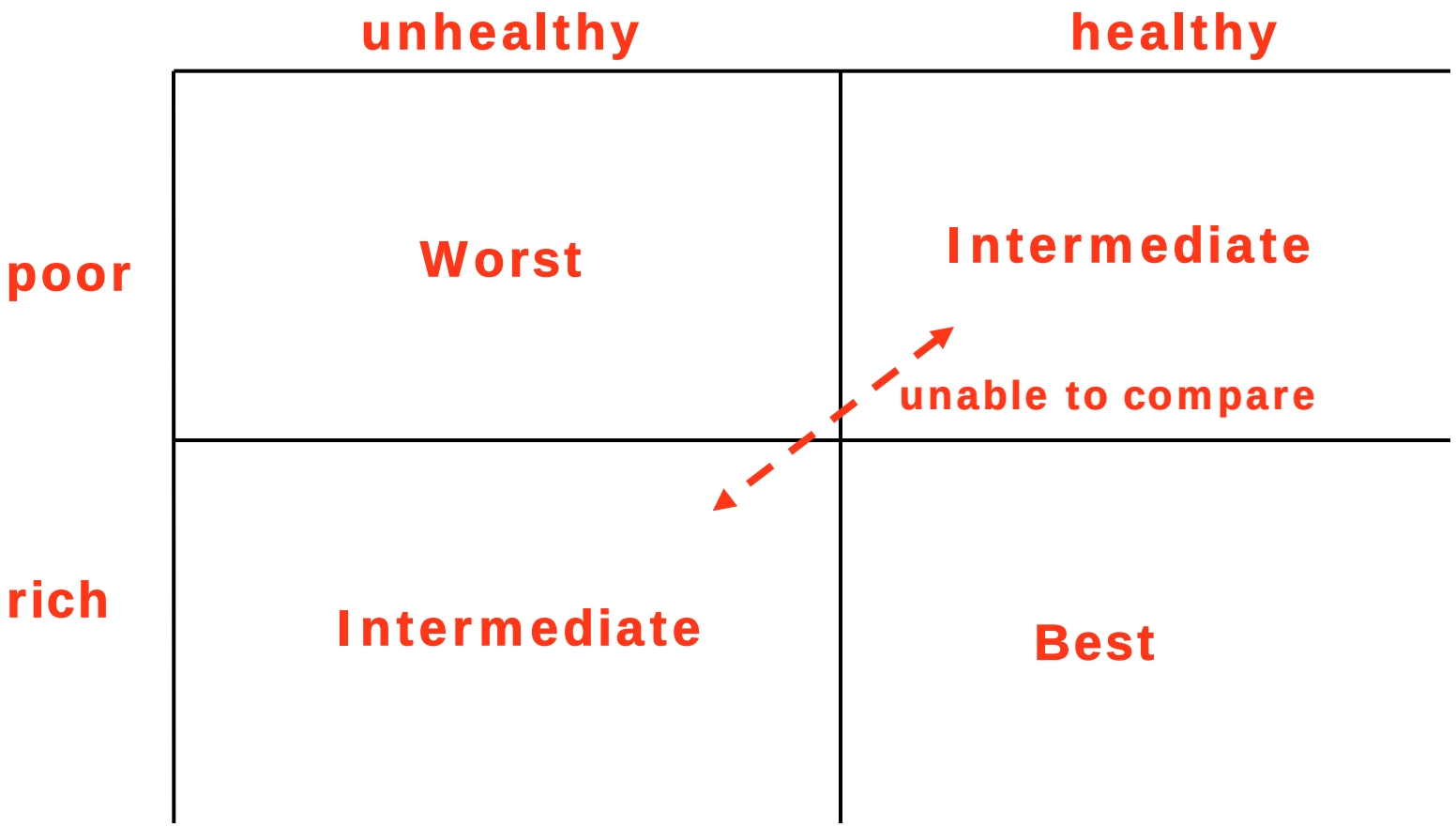


Illustration: 2 dimensions – 4 outcomes

	unhealthy	healthy
poor	Worst	Intermediate
rich	Intermediate	Best

Illustration: 2 dimensions – 4 outcomes



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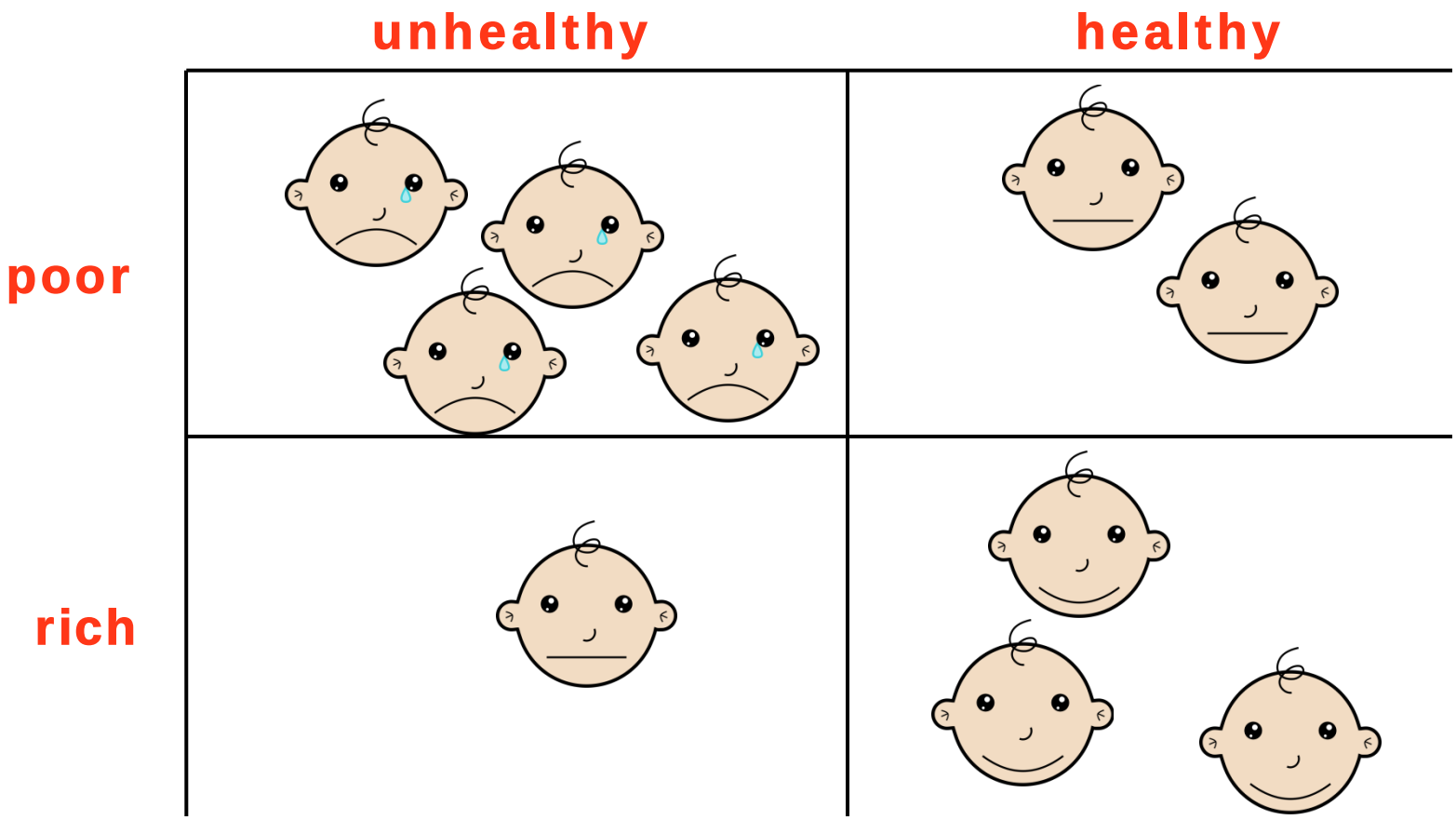

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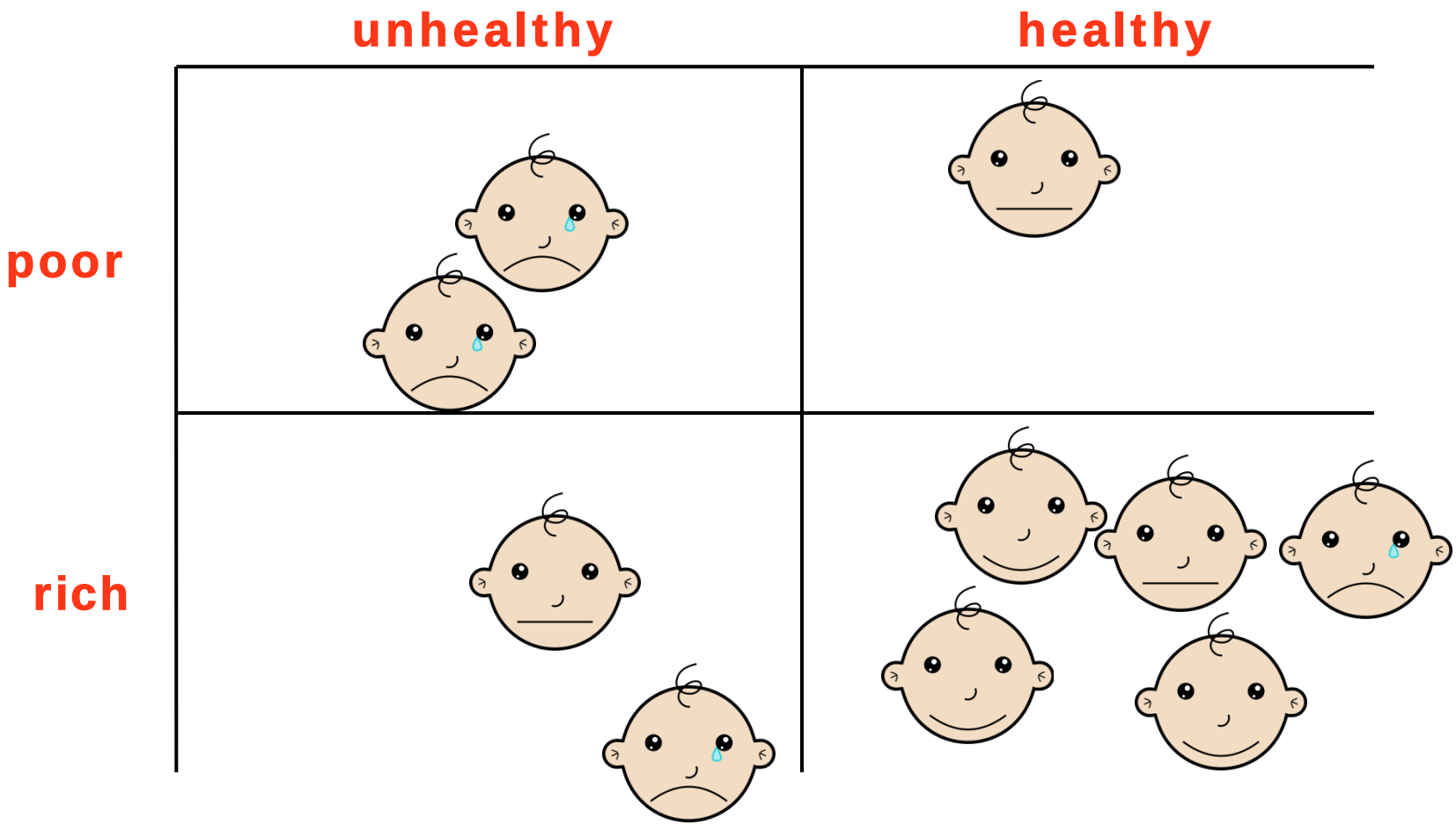


Generally, how can we check if B dominates A?

One can **compare the cumulative distributions over outcomes**. (For each lower comprehensive set of outcomes, the total mass at B should not be greater than at A).

or

Mosler and Scarcini (1991) and Dyckerhoff and Mosler (1997) observe that tests for first order dominance correspond to a **linear program**.

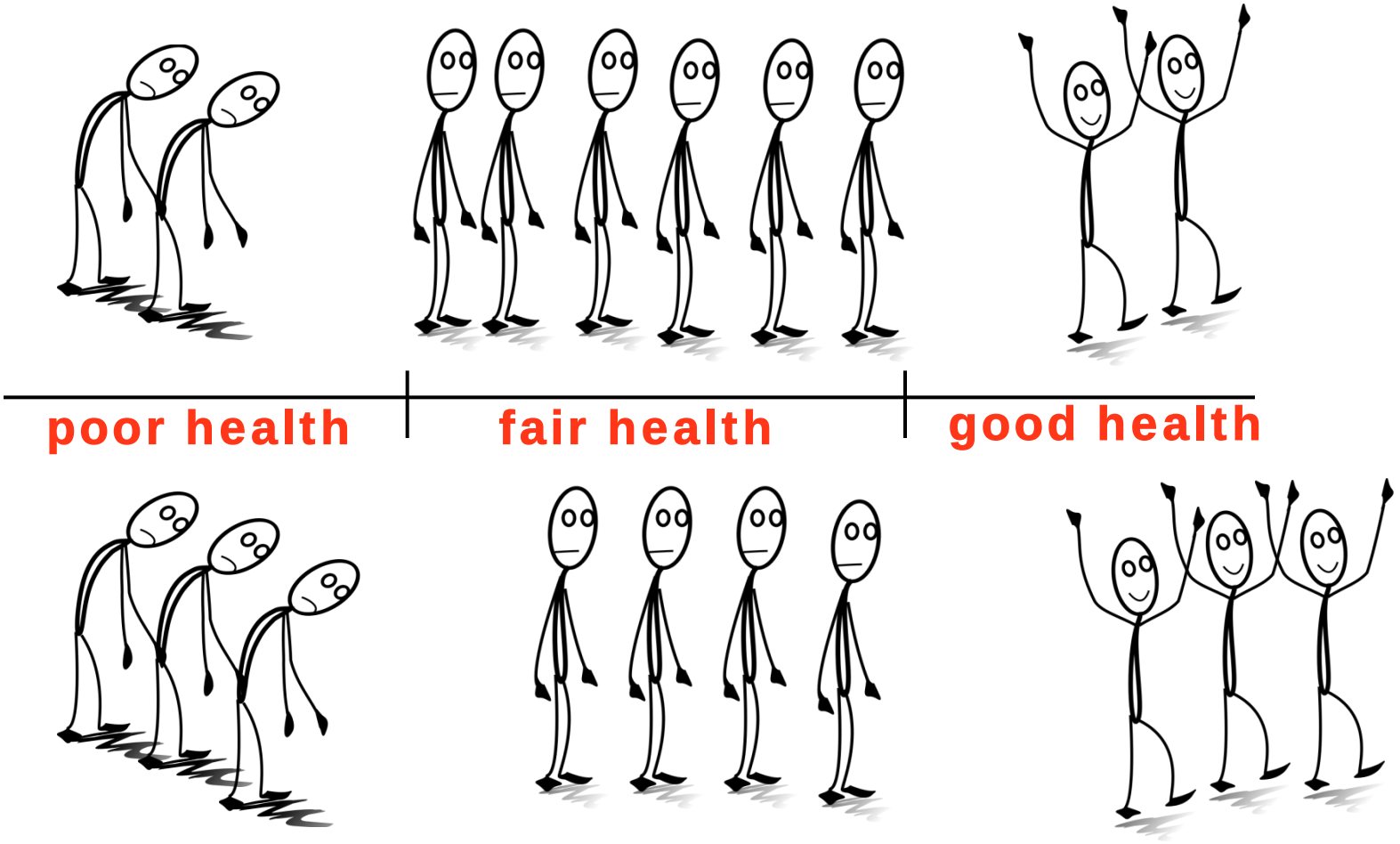
Arndt C et al (2012) **operationalize** the LP technique and apply it to multidimensional child deprivation comparisons in Mozambique and Vietnam.

Ordinal bivariate inequality comparisons

When is there more inequality **within** one group than another?

For the **one-dimensional case**, Allison and Foster (2004) put forward a simple but intuitive model for comparisons of inequalities when outcomes are ordinally ranked.

Which population has (health-wise) the most inequality?



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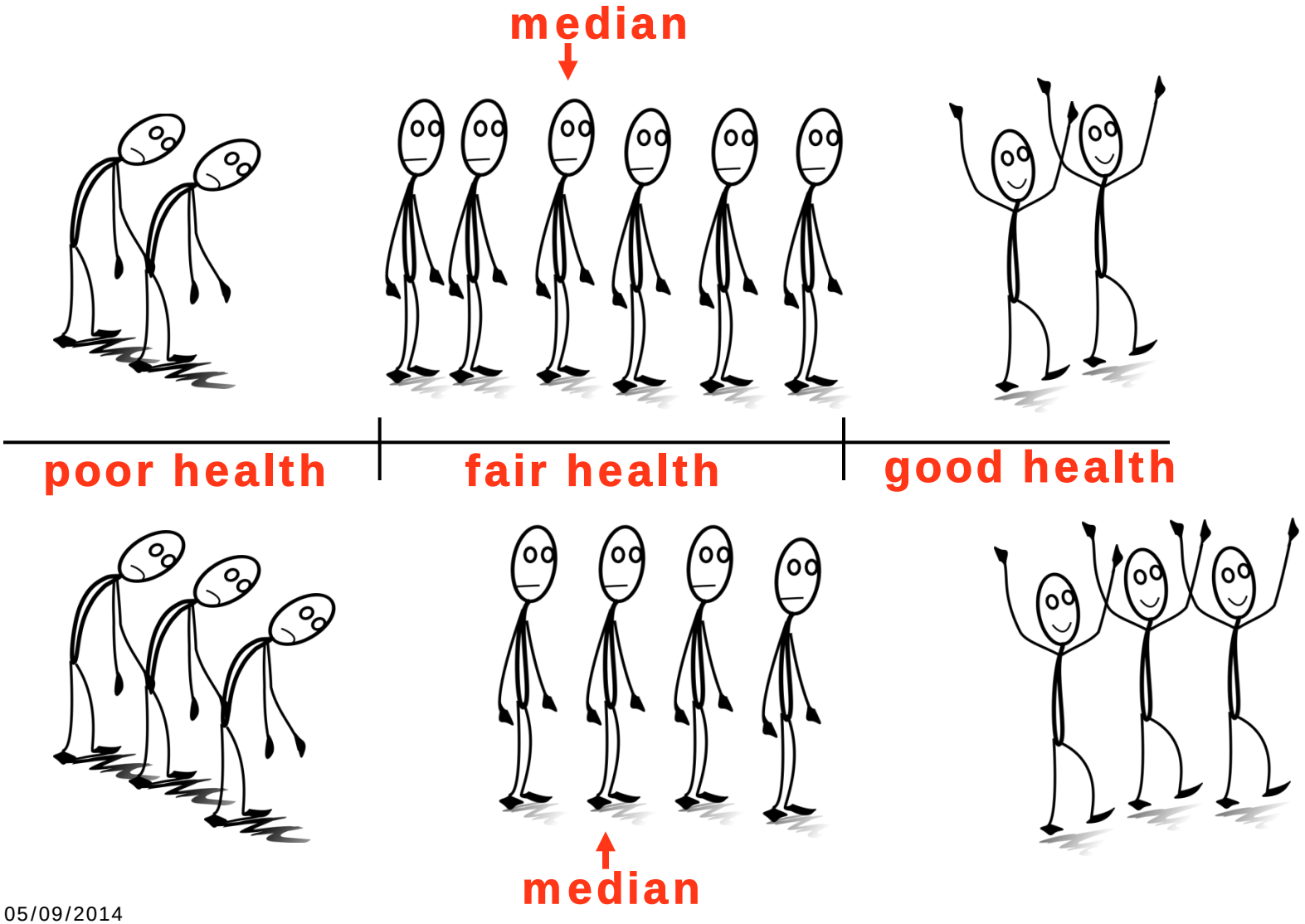

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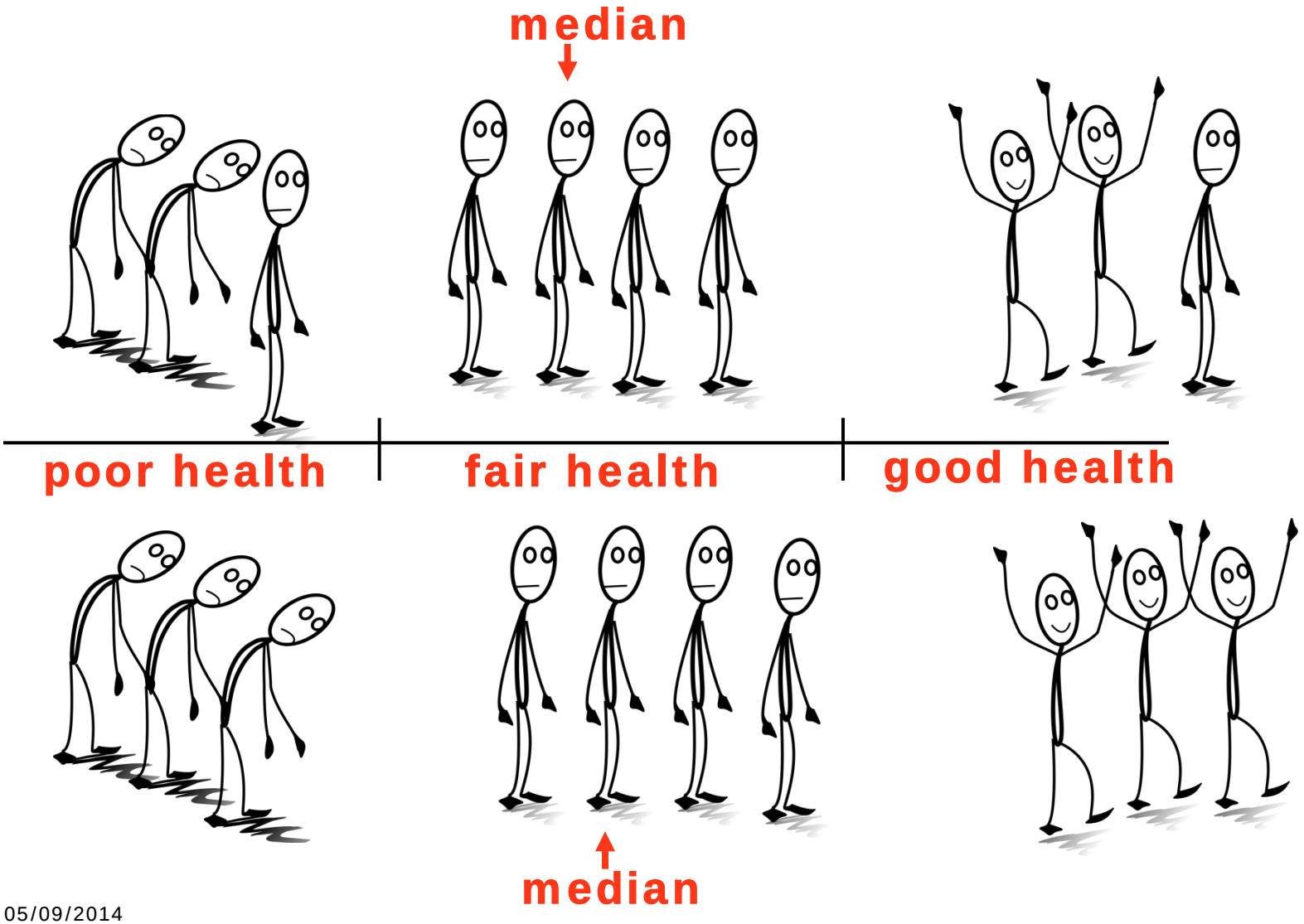
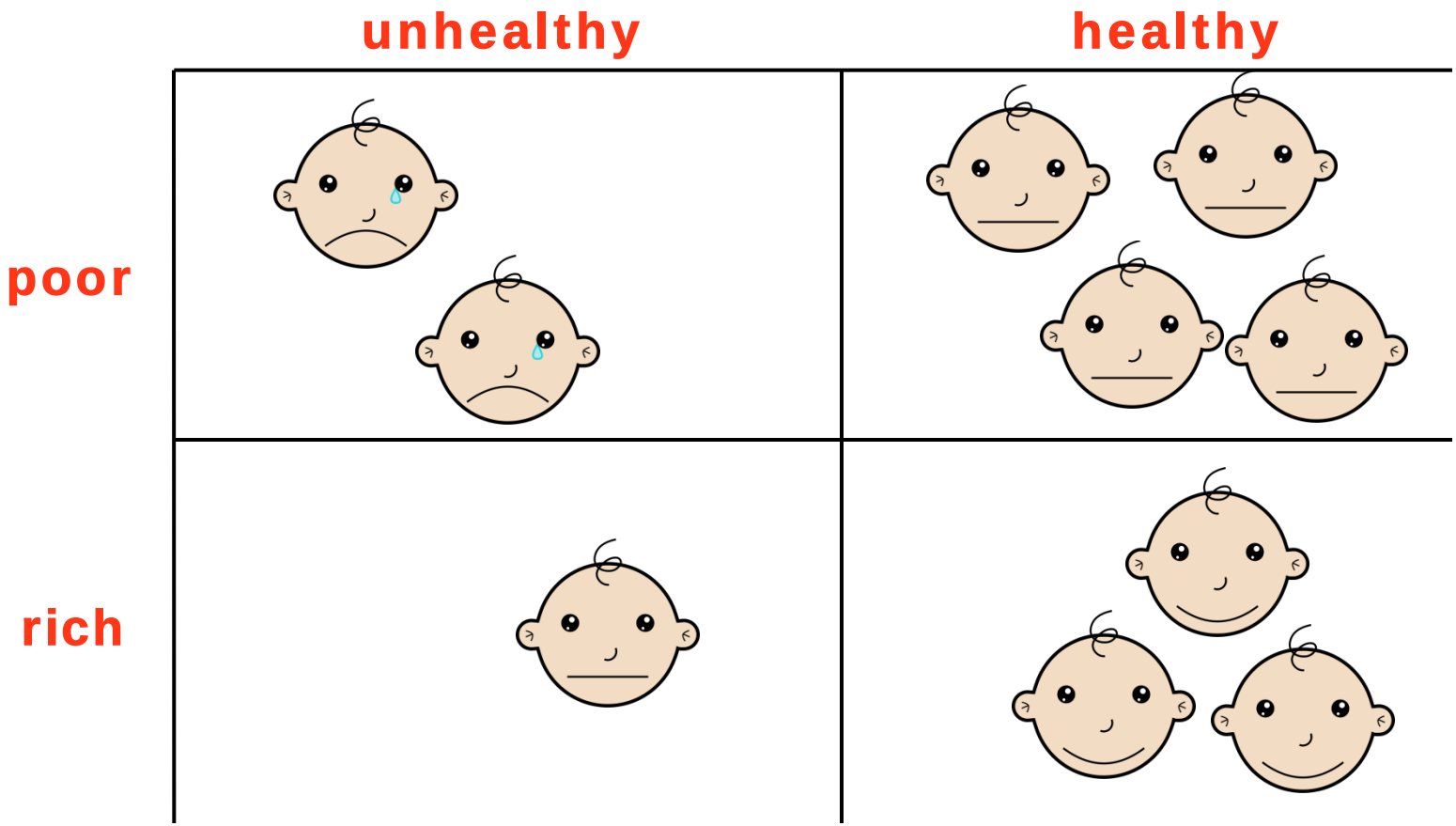


2 dimensions – 4 outcomes



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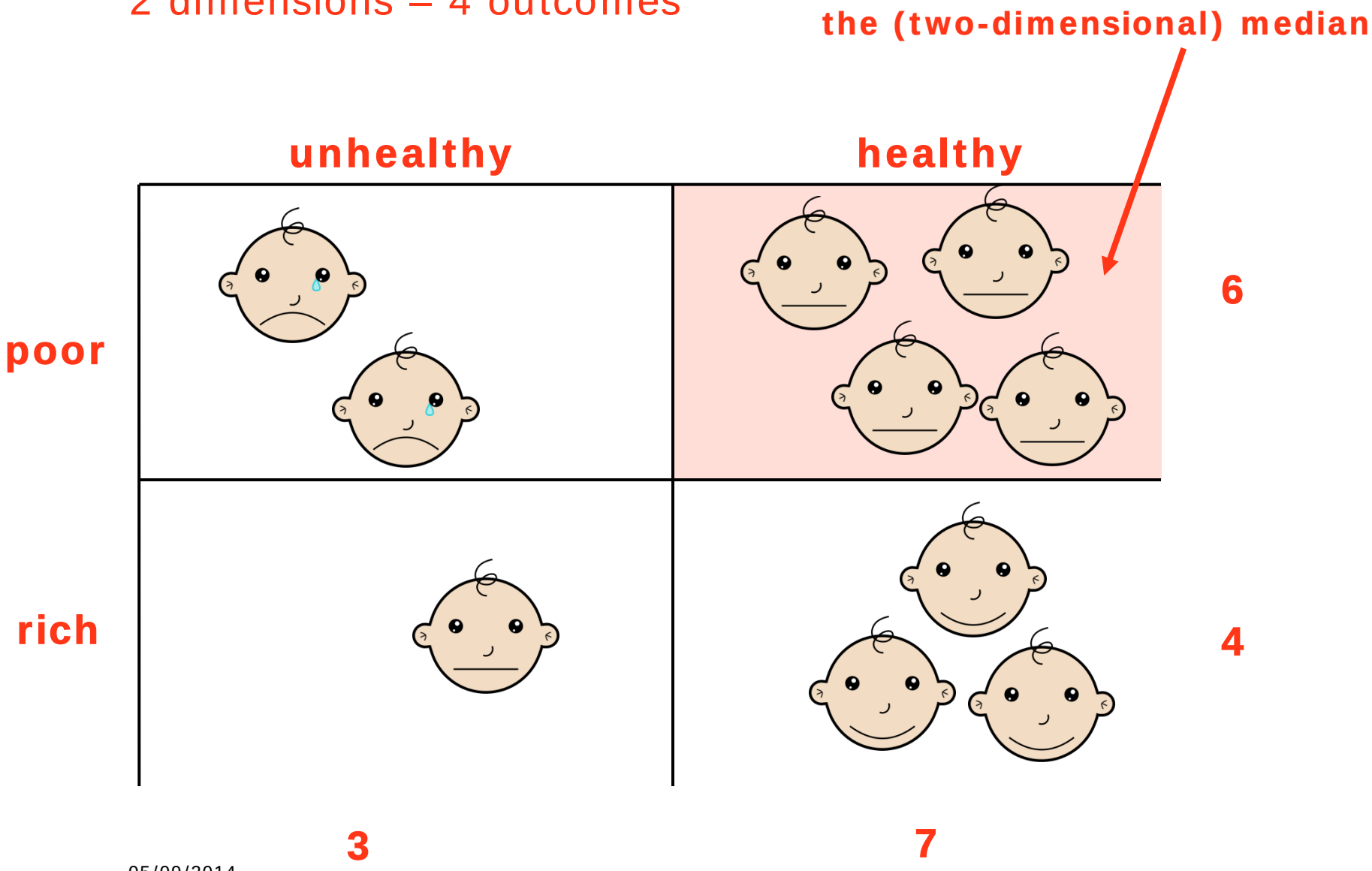

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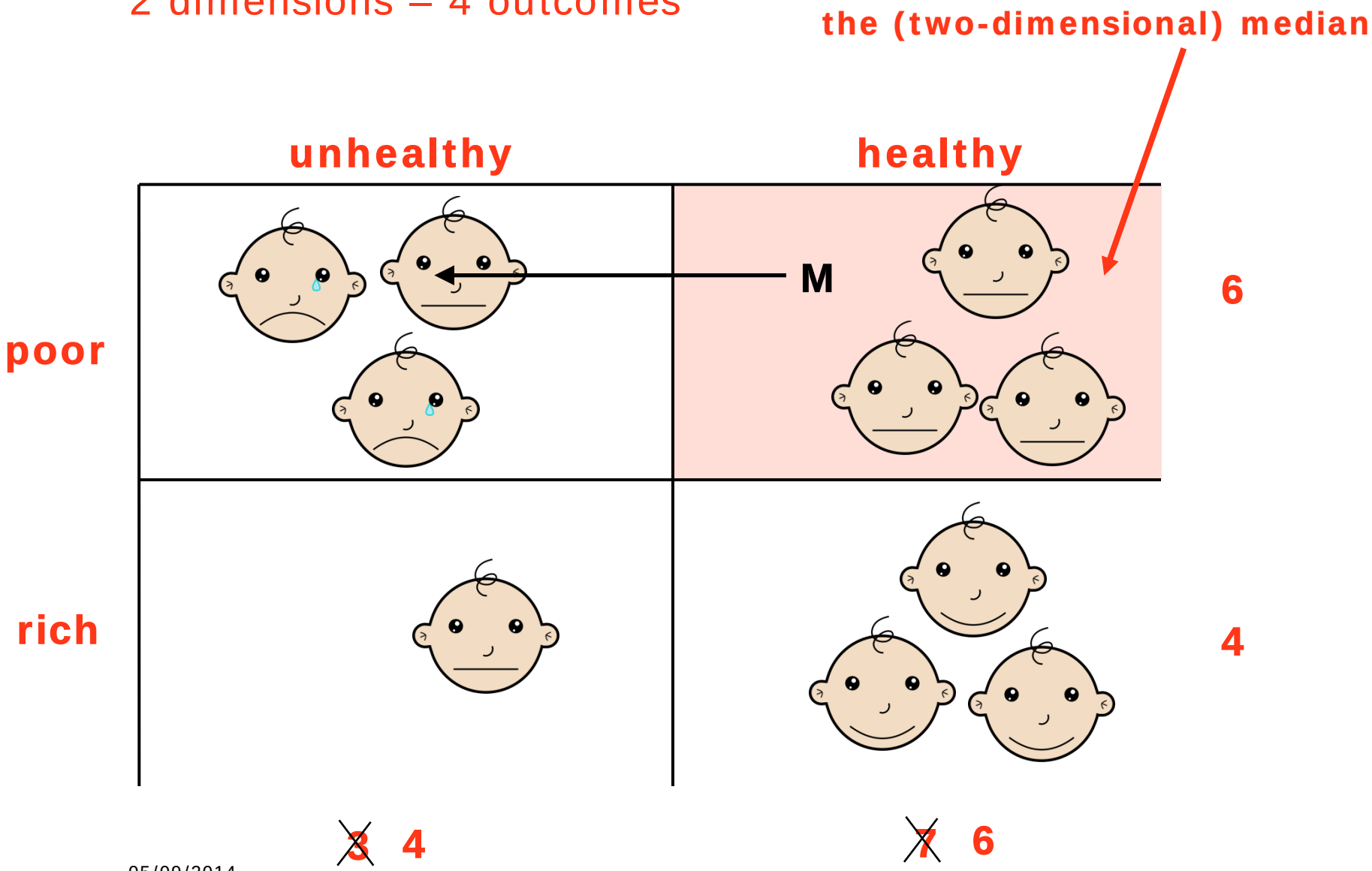

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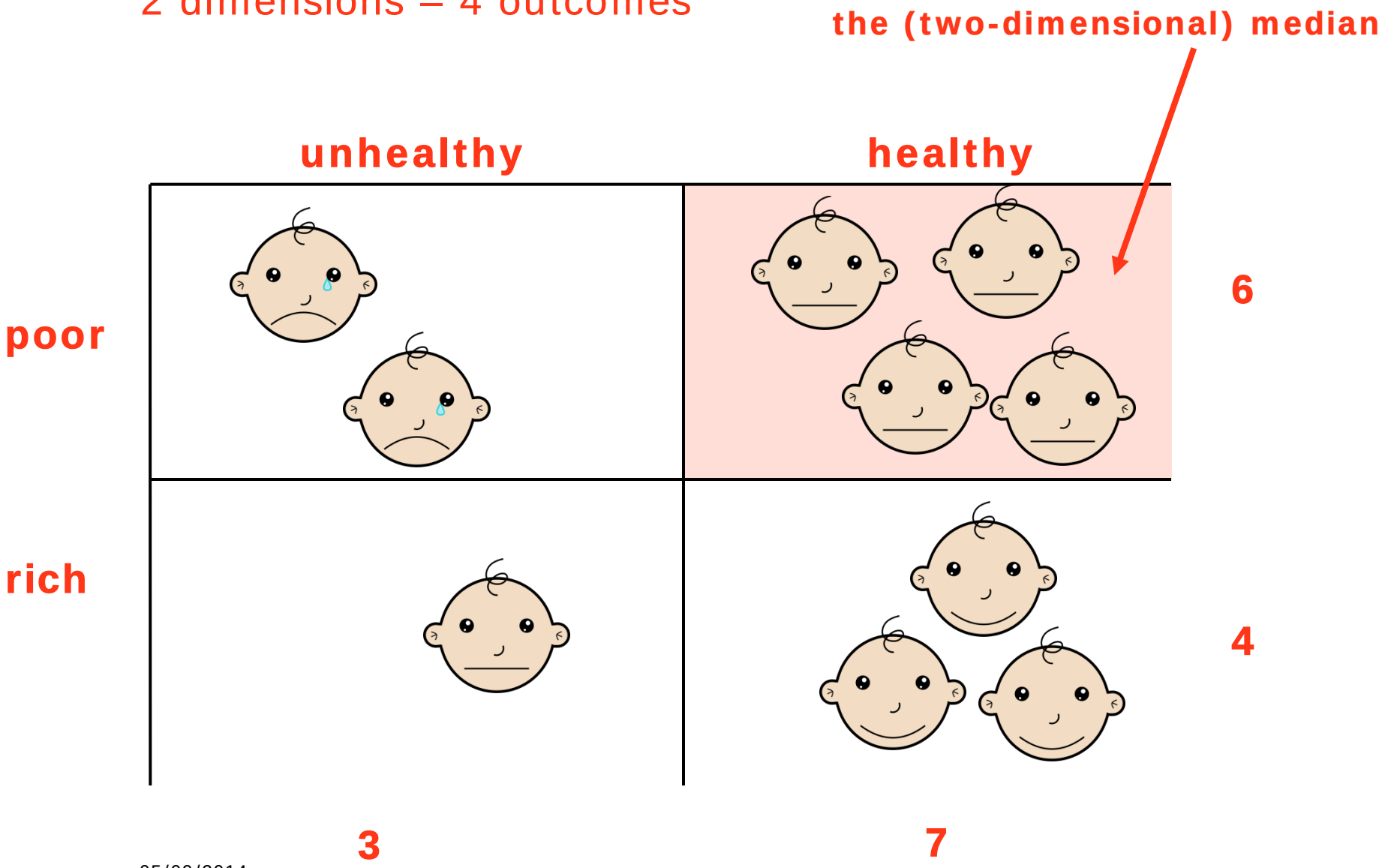

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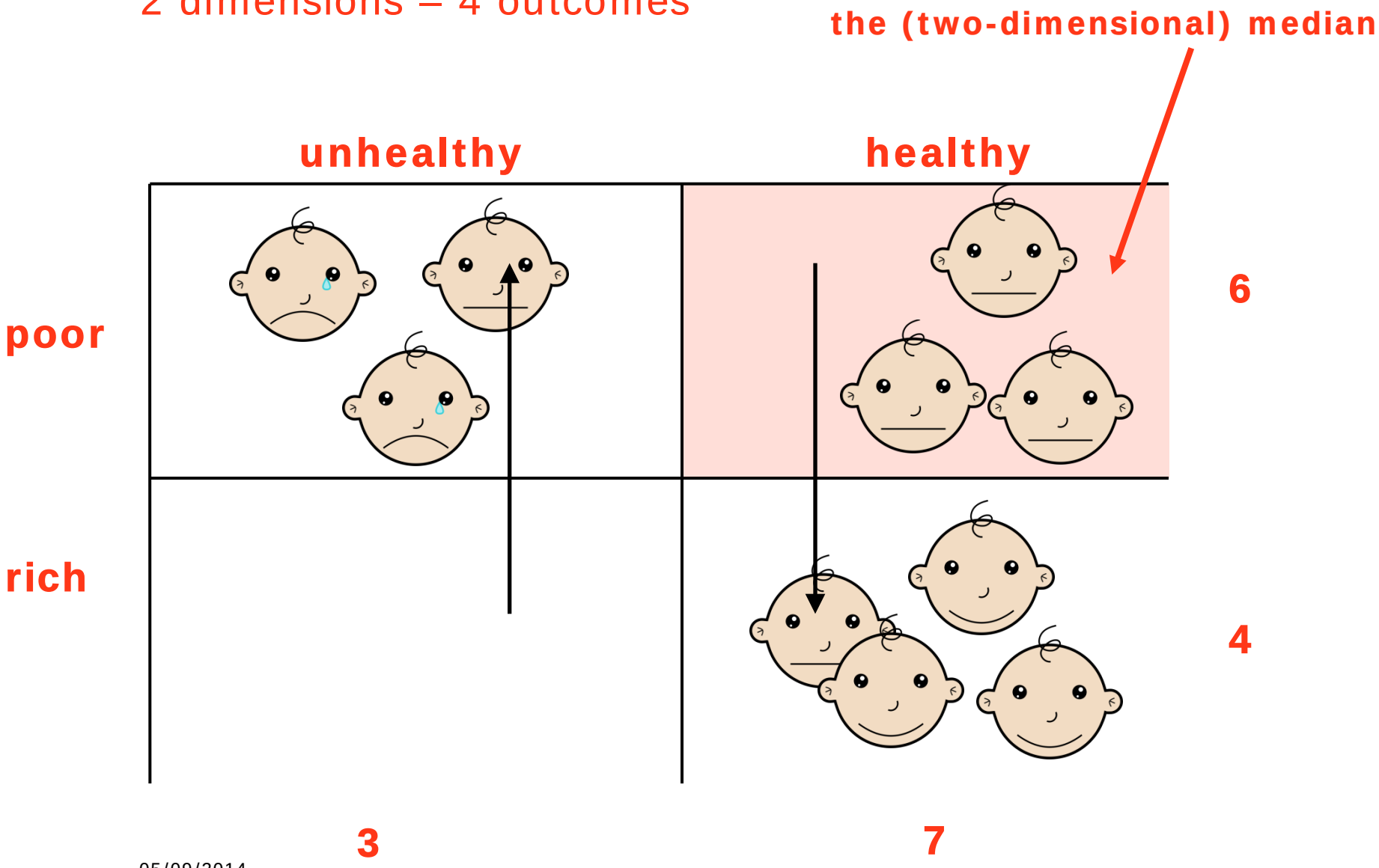
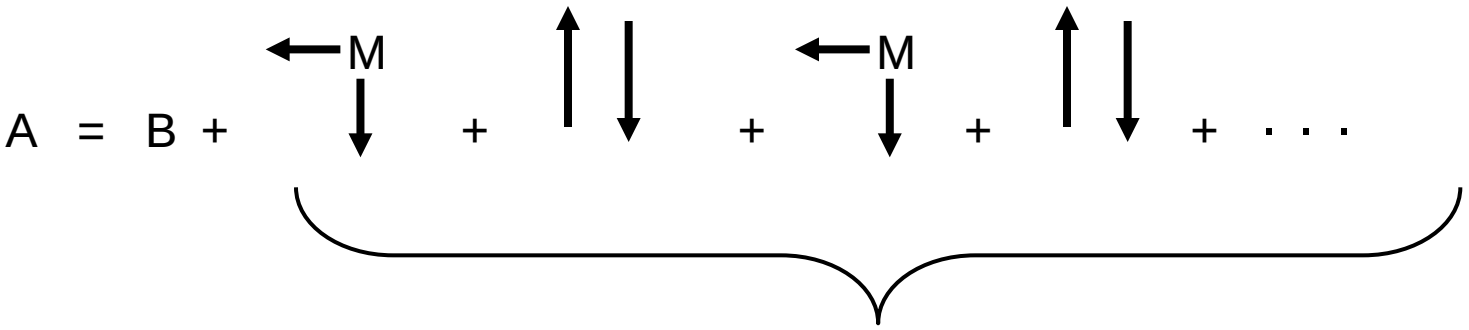


Ordinal multidimensional inequality concept

There is more inequality in population A than in population B if:

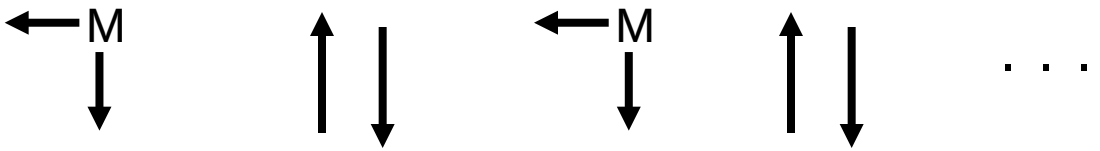


median-preserving spreads and correlation-increasing switches

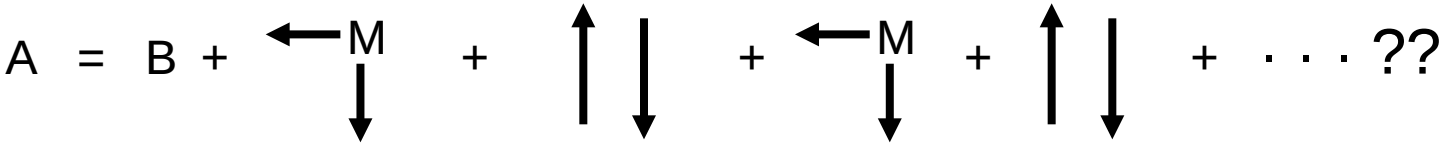
(inequality-increasing operations)

Checking for ordinal inequality.

Question: For populations A and B, when can we find some sequence



such that



Generally, a tricky problem. For the 2x2 case, we provide a set of necessary and sufficient (testable) conditions.

Child poverty in Mozambique (female headed rural households)

Severe health deprivation

Severe sanitation deprivation	Yes	No
Yes	13,9% / 15,7%	47,9% / 44,9%
No	4,7% / 3,7%	33,5% / 35,7%

Red: girls

Blue: boys

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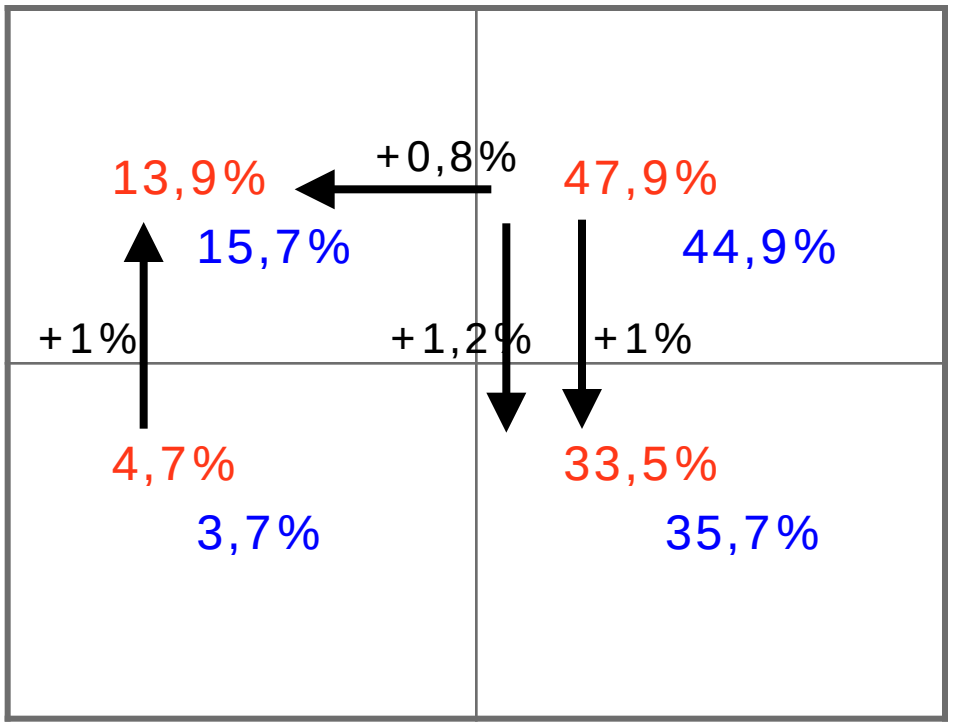

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Empirical application: child deprivation in Mozambique

Use 3 binary well-being indicators, so-called Bristol Indicators:

Severe sanitation deprivation (0-18 years)
Severe health care deprivation (0-5 years)
Severe education deprivation (7-18 years)

Further details: UNICEF (2006), Annex I.

Key characteristics:

Urban or **Rural** area of residence

Female or **Male** household head

Girl or **Boy**.

Total of 8 groups of children – all compared to each other.

Data obtained from the Mozambican DHS 2003.

Information is available for 33,058 children of less than 18 years of age.

(Sanitation deprivation, Education deprivation)

Area, Gender of household head, Gender of child	*Median*	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy
Rural, Male, Girl	*(0,1)*		0	0	0	0	0	0	0
Rural, Male, Boy	*(0,1)*	1*		1*	1*	0	0	0	0
Rural, Female, Girl	*(0,1)*	1*	0		0	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	0	0		0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*		0	0B*	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0B*
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0		0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	0	0	

(Sanitation deprivation, Health deprivation)

Rural, Male, Girl	*(0,1)*		1	0	0	0	0	0	0
Rural, Male, Boy	*(0,1)*	0		0	0	0	0	0	0
Rural, Female, Girl	*(0,1)*	1	1*		0C*	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	1*	0		0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*		0	0	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0		0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	0	0	

Note: The number 1 indicates that the row distribution first order dominates the column distribution. The letters B and C indicate that the row distribution is ordinally more equal of type B or C respectively cf. Proposition 1. We conducted tests of significance for first order dominance and ordinal inequality by using the permutation bootstrap method. * indicates a significant test statistic at the 5% level.

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Rural, Male, Boy	*(0,1)*	1*		1*	1*	0	0	0	0
Rural, Female, Girl	*(0,1)*	1*	0		0	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	0	0		0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*		0	0B*	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0B*
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0		0
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Rural, Male, Boy	*(0,1)*	0		0	0	0	0	0	0
Rural, Female, Girl	*(0,1)*	1	1*		0C*	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	1*	0		0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*		0	0	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0
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Rural, Female, Girl	*(0,1)*	1*	0		0	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	0	0		0	0	0	0
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Rural, Female, Girl	*(0,1)*	1	1*		0C*	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	1*	0		0	0	0	0
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Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0		0
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Rural, Male, Boy	*(0,1)*	0		0	0	0	0	0	0
Rural, Female, Girl	*(0,1)*	1	1*		0C*	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	1*	0		0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*		0	0	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0		0	0
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0		0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	0	0	

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Future research

Generalize the checking procedure (beyond the 2x2 case)

Other empirical applications

Explore alternative ordinal inequality concepts

Full paper is available at my personal webpage:

https://sites.google.com/site/lposterdal/

Email: lpro@sam.sdu.dk

Area, Gender of household head, Gender of child	*(0,0)*	*(0,1)*	*(1,0)*	*(1,1)*	*Median*	*# of obs.*
(Sanitation deprivation, Education deprivation)						
Rural, Male, Girl	27.7	34.5	10.3	27.6	*(0,1)*	3716
Rural, Male, Boy	16.3	41.3	9.4	33.0	*(0,1)*	4010
Rural, Female, Girl	21.6	38.4	8.7	31.2	*(0,1)*	1223
Rural, Female, Boy	19.2	41.0	8.1	31.7	*(0,1)*	1348
Urban, Male, Girl	6.0	9.9	7.2	76.9	*(1,1)*	2858
Urban, Male, Boy	5.0	13.1	5.3	76.6	*(1,1)*	2912
Urban, Female, Girl	8.2	9.0	5.3	77.5	*(1,1)*	1140
Urban, Female, Boy	7.2	11.2	4.2	77.4	*(1,1)*	1025
(Sanitation deprivation, Health deprivation)						
Rural, Male, Girl	18.8	44.4	4.8	32.0	*(0,1)*	2262
Rural, Male, Boy	19.2	44.8	4.7	31.3	*(0,1)*	2288
Rural, Female, Girl	13.9	47.9	4.7	33.6	*(0,1)*	580
Rural, Female, Boy	15.7	44.9	3.7	35.6	*(0,1)*	598
Urban, Male, Girl	2.6	18.8	7.6	71.0	*(1,1)*	1215
Urban, Male, Boy	2.9	18.2	8.1	70.9	*(1,1)*	1156
Urban, Female, Girl	2.0	19.5	3.2	75.3	*(1,1)*	382
Urban, Female, Boy	3.7	16.7	8.5	71.1	*(1,1)*	341

Note: The first element, i, in vector (i,j) indicates sanitation deprivation. The second element, j, indicates education deprivation in the top panel and health deprivation in the bottom panel. $i,j = 0$ is deprivation, $i,j = 1$ is no deprivation.

Source: Authors' calculations from DHS 2003.